THREE DIMENSIONAL FINITE - ELEMENT FORMULATION FOR FINLINE DISCONTINUITY PROBLEMS

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Abstract

A three dimensional finite - element formulation is proposed for finding directly the electric or magnetic field at a given frequency of excitation inside a general multiaxial finline discontinuity.

It is shown that this formulation can be used to find the scattering parameters of a general finline bend discontinuity problem. Results are presented for a case study of a finline step discontinuity problem.

Introduction

Theoretical investigations published so far for finline discontinuities deal with the step-discontinuity [1, 2], the inductive strip in finline [3] and the tapering [4]. In all these cases, the discontinuity results from the breaking down of the invariance of the waveguide ports during a space translation but the resulting ports of the electromagnetic system stay always uniaxial. But, to our knowledge, no attempt has been made to analyze mutiaxial finline discontinuities like the T junction and the general bend shown in Fig. 1.

The field theoretical solution of such discontinuities can find important applications in the precise analytical description of many passive circuit elements realized in finline technology like stubs, couplers [5], mixers [6] and modulators [7].

The complexity for efficient and accurate analysis of such discontinuities lies in the necessary treatment of hybrid eigen modes propagating in more than one direction and in the necessary description of fields inside the junction formed between the different system ports. The finite- element technique proved itself as the most adaptable one for facing such a situation.

This paper describes a three-dimensional finite-element formulation (FEF) for finding directly the electric or magnetic field at a given frequency of excitation inside a general multiaxial finline discontinuity. Also we describe how this formulation can be used to find the scattering parameters of a general multiport finline junction.

Due to the non-existence of available results for the characterisation of multiaxial finline discontinuities and owing to the capacity of our FEF to treat the uniaxial finline discontinuity as well as the multiaxial one, the validation of our analysis is performed through the comparison between the results of the characterisation of a finline impedance step problem obtained once by our FEF and another time by other analysis like that of [1].

Stationary Functional For the FEF

It may be shown [8] that the functional

$$F_E = \iiint_V [(\nabla \times \bar{E}).(\nabla \times \bar{E}) - \epsilon_r k^2 \bar{E}.\bar{E}]\, dV \quad (1)$$

(with $k^2 = \omega^2 \mu_0 \epsilon_0$) is stationary to perturbations about the true solutions of $\bar{E}$ satisfying Maxwell's equations within a volume V and conforming to appropriate boundary conditions on a surface S enclosing V. It is provided that E is constrained to conform the homogeneous Dirichlet condition $\bar{E} \times \bar{n} = 0$ on perfectly conducting walls and the homogeneous Neumann condition $(\nabla \times \bar{E}) \times \bar{n} = 0$ on magnetic walls.

Finite Element Discretisation

The volume V is broken into tetrahedral elements with ϵ_r and μ_r supposed constant within each element but with discontinuity between elements allowed. In each tetrahedron the electric field is approximated by a trial function complete to Mth order in the space co-ordinates, ie

$$\bar{E}(r) = \bar{E}^m \alpha^m (\xi) \quad (2)$$

$$\bar{r} = \bar{r}^m \alpha^m (\xi) \quad (3)$$

with $m = 1, \ldots, N = [(M+1)(M+2)(M+3)/6]$, $\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$ is the vector volume co-ordinates and $\alpha^m(\xi)$ is an interpolation polynomial which is constructed in the following manner

$$\alpha^m_{ijkl}(\xi) = P_i(\xi_1) P_j(\xi_2) P_k(\xi_3) P_l(\xi_4) \quad (4)$$

$$\text{with} \quad P_m(z) = \prod_{i=1}^{m} (Mz-i+1)/i \quad (5)$$

Substituting from equation (2) into equation (1) yields the following final expression for the functional for a single element.

$$F_{Ee} = {}^tE_{Ce}\, W_e\, E_{Ce} \quad (6)$$

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where E_{ce} is a column vector composed of cartesian components of the electric field at tetrahedron nodes and W_e is a square matrix

$$W_e = (QK - \varepsilon_r k^2 \delta T) V_e \qquad (7)$$

where δ is the kronecker delta and Q, T are universal matrices independent of the tetrahedron geometry and can be put in the form

$$Q_{mn}^{ij} = 6 \int_{\Omega} \frac{\partial \alpha_m}{\partial \xi_i} \frac{\partial \alpha_n}{\partial \xi_j} d\Omega \qquad (8)$$

$$T_{mn} = 6 \int_{\Omega} \alpha_m \alpha_n \, d\Omega \qquad (9)$$

The matrix K is a simple function of the tetrahedron vertex co-ordinates given by

$$K_{ij}^{st} = \delta_{st} \frac{\partial \xi_i}{\partial r_w} \frac{\partial \xi_j}{\partial r_w} - \frac{\partial \xi_i}{\partial r_s} \frac{\partial \xi_j}{\partial r_t} \qquad (10)$$

where s, t, w are cartesian axis labelling.

Summing up for all the elements e, partitionning the field E into E_F, which is a column vector representing all of the free components of E at the nodes, and E_P, which represents the prescribed tangential components that are either zero (on perfectly conducting walls) or known, the global functional takes the form

$$F_E = {}^tE_p W_{pp} E_p + {}^tE_p W_{pF} E_F + {}^tE_F W_{Fp} E_p + {}^tE_p W_{FF} E_F \qquad (11)$$

Finally making F_E ($F_E = \sum F_{Ee}$) stationary with respect to all variations of vector E_F leads to the linear matrix equation

$$W_{FF} E_F = - W_{Fp} E \qquad (12)$$

which can be solved for the unknowns E_F

Application to a Finline Bend Discontinuity

Fig. 2 shows a general bend discontinuity in a finline. The scattering matrix S for the junction region with respect to reference planes J_1 and J_2 is defined as

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad (13)$$

where a_1, b_1, a_2, b_2 are the wave amplitudes of forward and reverse dominant-mode waves in the two finline ports. It is supposed that the transverse electric fields E_t are precisely prescribed at planes P_1 and P_2 sufficiently far from the bend so that the E_t values can be extracted from a knowledge of the field components of the dominant-mode propagating on a uniform finline [1]. The field solution is then determined everywhere between planes P_1 and P_2 using (12). Numerical processing of the field amplitudes near P_1 and P_2 will determine the ratios $R_1 = b_1/a_1$ and $R_2 = b_2/a_2$. A little algebraic manipulation of equation (13) gives

$$S_{21} S_{12} = (R_1 - S_{11})(R_2 - S_{22}) \qquad (14)$$

If the dielectric substrate is isotropic, then $S_{12} = S_{21}$. Repeating the process for other two positions for the planes P_1, P_2 allows S_{11}, S_{22} and S_{12} to be determined.

Case Study

In order to evaluate objectively the effectiveness of the FEF, it is applied to the solution of a finline step discontinuity problem as that shown in Fig. 3. A general finite-element program MODULEF [9] has been used to implent numerical method. Figure 4 shows the finite element model for the discontinuity problem.

The matrix equation (12) is solved by the Gauss-Seidel method.

It worths mentionning that the plane P_2 is chosen to be a short circuit one. Figure 5 shows examples of the results that show the variations of the different transverse electric field components as functions of the coordinates at the shown positions (x_0, y_0, z_0) between planes P_1 and P_2.

Furthermore, Table 1 gives the values of the scattering matrix coefficients for the dominant mode for the case study at 30 GHz. The shown values agrees well with these obtained from the modal analysis [1].

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TABLE 1

Frequency = 30 GHz

β_1 (Propagation constant for dominant mode) for W_1 = 1 mm : 598.3 rad/m

β_2 (Propagation constant for dominant mode) for W_2 = 2 mm : 535.18 rad/m

POSITION	Z_1 (mm)	Z_2 (mm)	R_1	R_2
1	−10	10	− 0.728 − j 0.686	0.288 + J 0.958
2	−9	9	− 0.841 + j.540	− 0,702 + j 0.712
3	−11	11	− 0.0421 − j 0.999	0.978 + j 0.207

➡ S_{11} = 0.0804 − j 0.261

S_{12} = 0.652 + j 0.706

S_{22} = 0.253 + j 0.102

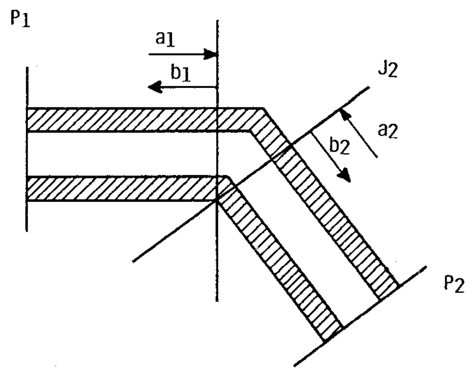


FIG. 2 REFERENCE PLANES FOR THE BEND

P_1, P_2 référence planes for prescribed fields.

J1, J2 référence planes for the scattering matrix.

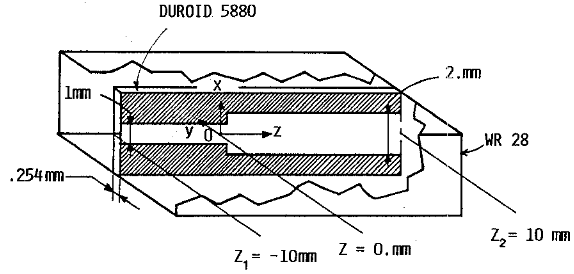
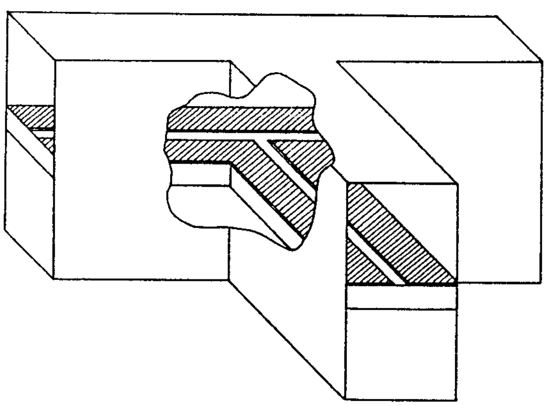


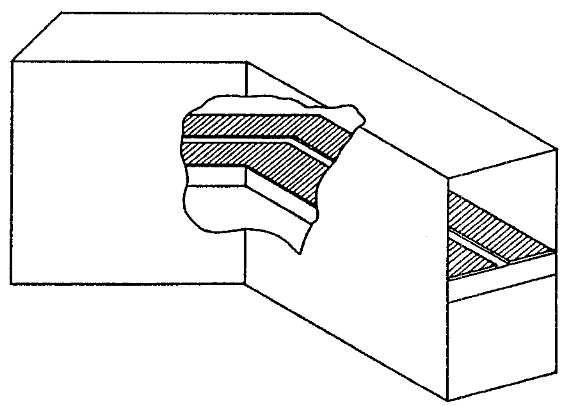
FIG. 3 IMPEDANCE STEP DISCONTINUITY IN FINLINE TECHNOLOGY.

The transverse prescribed field at the plane z=-10 mm is the fundamental propagating finline mode derived from [1]

The transverse plane at z = 10 mm is conducting (short circuit)

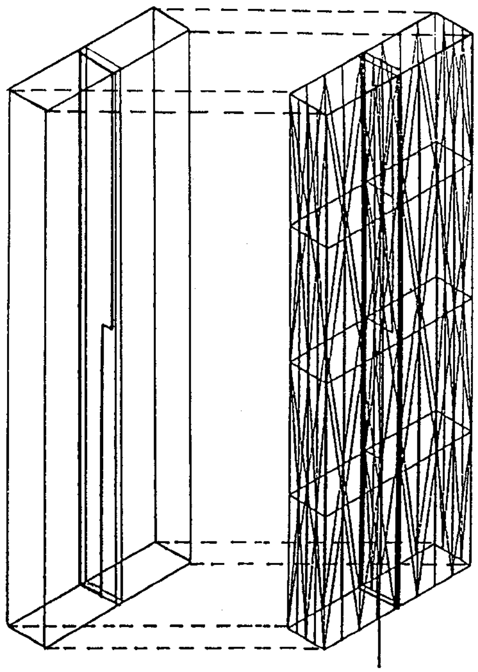


a) T–JUNCTION IN FINLINE TECHNOLOGY

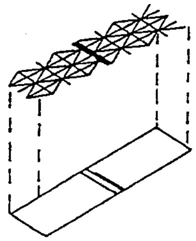


b) BEND IN FINLINE TECHNOLOGY

Figure 1 : 3–D FINLINE DISCONTINUITIES



a) Longitudinal



b) Transverse

FIG. 4 MESH DESCRIPTION OF THE FINLINE IMPEDANCE STEP

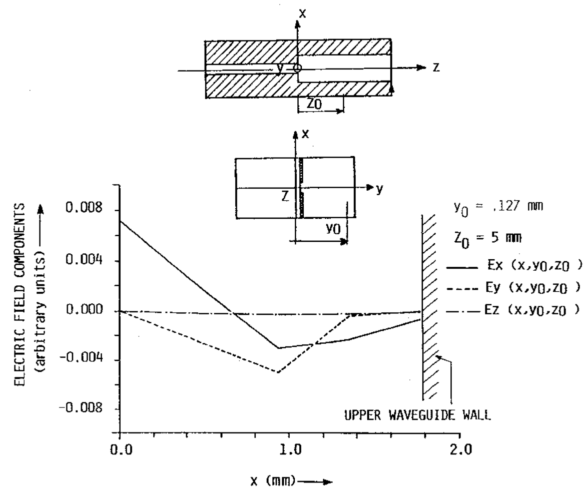


FIG.5a FINITE -ELEMENT FIELD RESULTS

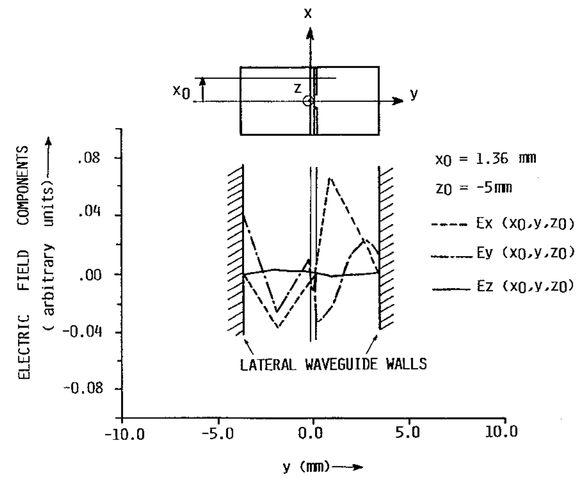


FIG. 5b FINITE-ELEMENT FIELD RESULTS

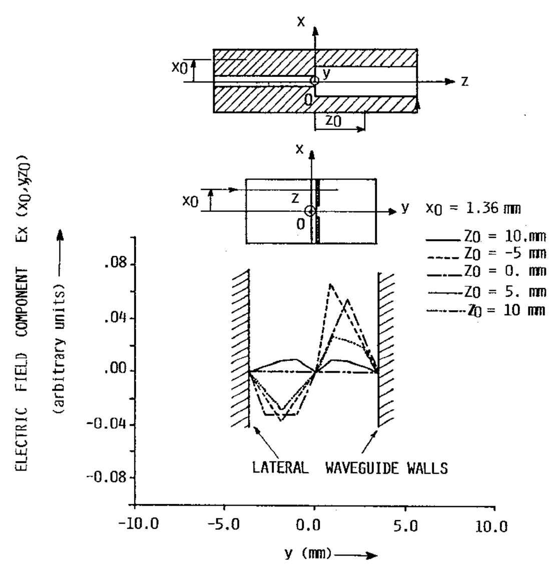


FIG.5c FINITE-ELEMENT FIELD RESULTS